

ii. If molar volume is the volume occupied by 1 mole of any (ideal) gas at STP, show that it is 22.4 L (take $R = 8.313 \text{ J mol}^{-1} \text{ K}^{-1}$).

OR

When do the real gases obey more correctly the gas equation : $PV = nRT$?

24. The position of an object moving along the x-axis is given by $x = a + bt^2$, where $a = 8.5 \text{ m}$, $b = 2.5 \text{ m}$ and t is measured in seconds. What is its velocity at $t = 0 \text{ s}$ and $t = 2.0 \text{ s}$? What is the average velocity between $t = 0 \text{ s}$ and $t = 4.0 \text{ s}$? [2]

25. A constant retarding force of 50 N is applied to a body of mass 30 kg moving initially with a speed of 18 ms^{-1} . How long does the body take to come to a halt? [2]

Section C

26. What is a cyclic process? What is change in internal energy of the system in a cyclic process? In changing the state of a gas adiabatically from an equilibrium states A to B, an amount of 40.5 J of work is done on the system. If the gas is taken from states A to B via a process in which net heat absorbed by the system is 12.6 cal. How much is the net work done by the system in the later case? ($1 \text{ cal} = 4.19 \text{ J}$) [3]

27. A 100 kg gun fires a ball of 1 kg horizontally from a cliff of height 500 m. It falls on the ground at a distance of 400 m from the bottom of the cliff. Find the recoil velocity of the gun. (acceleration due to gravity = 10 ms^{-2}). [3]

28. Two mercury droplets of radii 0.1 cm. and 0.2 cm. collapse into one single drop. What amount of energy is released? The surface tension of mercury $T = 435.5 \times 10^{-3} \text{ N m}^{-1}$. [3]

OR

What is venturi-meter? On which principle does it work? How is the principle of venturi-meter applied in automobiles?

29. The two individual wave functions are $y_1 = 5 \sin (4x - t) \text{ cm}$ and $y_2 = 5 \sin (4x + t) \text{ cm}$ where, x and y are in centimeters. Find out the maximum displacement of the motion at $x = 2.0 \text{ cm}$. Also, find the positions of nodes and antinodes. [3]

OR

Earthquakes generate sound waves inside the earth. Unlike a gas, the earth can experience both transverse (S) and longitudinal (P) sound waves. Typically the speed of S wave is about 4.0 km s^{-1} , and that of P wave is 8.0 km s^{-1} . A seismograph records P and S waves from an earthquake. The first P wave arrives 4 min before the first S wave. Assuming the waves travel in straight line, at what distance does the earthquake occur?

30. A 10 kW drilling machine is used to drill a bore in a small aluminium block of mass 8.0 kg. How much is the rise in temperature of the block in 2.5 minutes, assuming 50% of power is used up in heating the machine itself or lost to the surroundings. Specific heat of aluminium = $0.91 \text{ J g}^{-1} \text{ K}^{-1}$. [3]

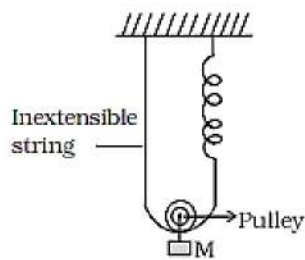
Section D

31. A tunnel is dug through the centre of the Earth. Show that a body of mass ' m ' when dropped from rest from one end of the tunnel will execute simple harmonic motion. [5]

OR

Find the time period of mass M when displaced from its equilibrium position and then released for the system shown in figure.

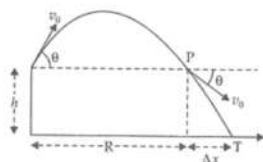




32. A marble rolls along a table at a constant speed of 1.00 m/s and then falls off the edge of the table to the floor 1.00 m below,
- How long does the marble take to reach the floor?
 - At what horizontal distance from the edge of the table does the marble land?
 - What is its velocity as it strikes the floor?

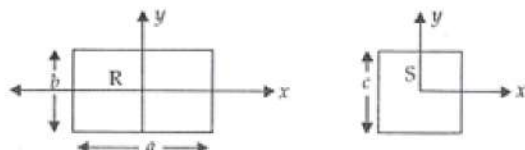
OR

A gun can fire shells with maximum speed v_0 and the maximum horizontal range that can be achieved is $R = \frac{v_0^2}{g}$. If a target farther away by distance Δx (beyond R) has to be hit with the same gun as shown in the figure here, show that it could be achieved by raising the gun to a height at least $h = \Delta x \left[1 + \frac{\Delta x}{R} \right]$



[Hint: This problem can be approached in two different ways:

- Refer to the diagram: target T is at the horizontal distance $x = R + \Delta x$ and below the point of projection $y = -h$.
 - From point P in the diagram: Projection at speed v_0 at an angle θ below horizontal with height h and horizontal range Δx .]
33. A uniform square plate S (side c) and a uniform rectangular plate R (sides b, a) have identical areas and masses [5] (Figure).



Show that

- $\frac{I_{xR}}{I_{xS}} < 1$
- $\frac{I_{yR}}{I_{yS}} > 1$
- $\frac{I_{zR}}{I_{zS}} > 1$

OR

- Find the moment of inertia of a sphere about a tangent to the sphere, given the moment of inertia of the sphere about any of its diameters to be $\frac{2MR^2}{5}$, where M is the mass of the sphere and R is the radius of the sphere.
- Given the moment of inertia of a disc of mass M and radius R about any of its diameters to be $\frac{MR^2}{4}$, find its moment of inertia about an axis normal to the disc and passing through a point on its edge.

Section E

34. Read the text carefully and answer the questions: [4]
- Free fall is a kind of motion that everybody can observe in daily life. We drop something accidentally or purposely and see its motion. At the beginning its speed is zero and until the end it gains speed and before it



reaches ground its maximum speed. It gains speed approximately 10 m/s in a second while falling because of the gravitation.

During the fall, the air resistance is neglected and the acceleration remains constant (equal to g). The object is said to be in free fall. If the height through which the object falls is small compared to the earth's radius, g can be taken to be constant and equal to 10 m/s^2 approximately.

If the object is dropped from the top of a tall building, and it takes t seconds to reach the ground then the velocity when it reaches ground is gt . The height of the building is $\frac{1}{2}gt^2$.

- (i) If an object dropped from the top of a tall building takes 2 seconds to reach ground, find the height of the building?
- (ii) Which assumptions are considered when object falls freely from height?
- (iii) Draw velocity time graph of an object during free fall.

OR

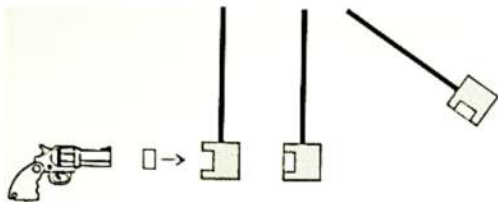
Draw acceleration-time graph during free fall.

35. **Read the text carefully and answer the questions:**

[4]

The ballistic pendulum was invented in 1742 by English mathematician Benjamin Robins.

A Ballistic Pendulum is a device for measuring a bullet's momentum and speed by employing perfectly inelastic collision.



A large wooden block suspended by two cords serves as the pendulum bob. When a bullet is fired into the bob, it gets embedded in the bob and its momentum is transferred to the bob.

The bullet's momentum and velocity can be determined from the amplitude of the pendulum swing. The velocity of the bullet, in turn, can be derived from its calculated momentum.

After collision, if the pendulum reaches a height h , then from principle of conservation of mechanical energy

$$\frac{1}{2}(m + M)v_p^2 = (m + M)gh$$

where, m = mass of bullet, M = mass of the bob v_p = velocity of the bob-bullet combination

$$\therefore v_p = \sqrt{2gh}$$

Now, Momentum before collision = Momentum after collision

$$mv_B = (m + M)v_p$$

where, v_B = velocity of bullet

$$v_B = \frac{m+M}{m} \sqrt{2gh}$$

the ballistic pendulum used to be a common tool for the determination of the muzzle velocity of bullets as a measure of the performance of firearms and ammunition (Nowadays, the ballistics pendulum has been replaced by the ballistic chronograph, an electronic device).

- (i) What type of collision occurs in ballistic pendulum and why?
- (ii) Which two principles of Physics are applied to find the velocity of the bullet?
- (iii) Ballistic pendulum has been replaced by which electronic device?

OR

A ballistic pendulum of 1 kg is fired with a bullet of mass 1 g. If the pendulum rises 2 cm, find the velocity of the bullet.

Solution
SAMPLE PAPER - 1
Class 11 - Physics
Section A

1. (d) 10^{-7}

Explanation: $[P] = [M^1L^{-1}T^{-2}]$

$$\begin{aligned} [1 \text{ lacsap}] &= [1 \text{ kg}] [100 \text{ km}]^{-1} [10 \text{ s}]^{-2} \\ &= 1 \text{ kg} \times 10^{-5} \text{ m}^{-1} \times 10^{-2} \text{ s}^{-2} \\ &= 10^{-7} \text{ kg m}^{-1} \text{ s}^{-2} = 10^{-7} \text{ Pa} \end{aligned}$$

2. (a) Viscous force

Explanation: Viscous force is a non-conservative force.

3. (a) $\frac{7}{10}mv^2$

Explanation: The total kinetic energy of a solid sphere rolling on the surface is given by:

$$\begin{aligned} \text{KE} &= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \\ &= \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right) \times \left(\frac{v}{R}\right)^2 \\ &= \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \frac{7}{10}mv^2 \end{aligned}$$

4. (b) 2.3

Explanation: 2.3

5. (b) $R_e\omega^2$

Explanation: Acceleration due to gravity at a place of latitude λ due to rotation of earth is $g' = g - R_e\omega^2 \cos^2 \lambda$

At equator, $\lambda = 0$, $\cos 0 = 1$

$$\therefore g' = g_e = g - R_e\omega^2$$

At poles, $\lambda = 90$, $\cos 90 = 0$

$$\therefore g' = g_p = g$$

$$\therefore g_p - g_e = g - (g - R_e\omega^2) = R_e\omega^2$$

6. (a) $\frac{3}{T}$

Explanation: $pT^2 = \text{constant}$

$$\therefore \left(\frac{nRT}{V}\right) T^2 = \text{constant or } T^3V^{-1} = \text{constant}$$

Differentiating the equation, we get

$$\frac{3T^2}{V} \cdot dT - \frac{T^3}{V^2}dV = 0 \quad \text{or} \quad 3dT = \frac{T}{V} \cdot dV \quad \dots(i)$$

From the equation

$$dV = V_\gamma dT$$

$$\gamma = \text{coefficient of volume expansion of gas} = \frac{dV}{V \cdot dT}$$

$$\text{From Eq. (i)} \quad \gamma = \frac{dV}{V \cdot dT} = \frac{3}{T}$$

7. (c) A thermometer should have a small thermal capacity

Explanation: To measure the temperature of a body, the thermometer is kept in contact with the body and heat transfer from the body to thermometer takes place. If the thermometer has a large thermal capacity, then the temperature of the body may change significantly. To minimise the change in temperature of the body, the thermal capacity of the thermometer is kept very low. According to Einstein, mass and energy are interconvertible. If Δm mass is lost then the energy released will be equal to $E = (\Delta m)c^2$. But the total energy of the universe is constant. Therefore, it should be said that the total mass and energy of the universe is conserved. Heat energy alone is not conserved.

When the temperature of the atmosphere tries to increase the evaporation of water from the sea surface takes place. Therefore, it permits a little change in the temperature. Hence, the temperature in cities on the coast changes very little.

8. (c) $\frac{v}{\lambda}$

Explanation: Let the two sources A, B emit sound of frequency $\nu_0 = \frac{v}{\lambda}$

When observer moves $B \rightarrow A$,

It receives compressed waves from A, whose frequency is:

$$v_A = v_o \left(\frac{u+v}{u} \right) \text{ [From Doppler's Effect]}$$

It receives elongated waves from B, given by

$$v_B = v_o \left(\frac{u-v}{u} \right)$$

Thus, frequency of beats observed by him:

$$v_{\text{beat}} = |v_A - v_B| = v_o \left(\frac{u+v - u+v}{u} \right)$$

$$\Rightarrow v_{\text{beat}} = \frac{2v^2}{u\lambda} \left(asv_o = \frac{v}{\lambda} \right)$$

9. (b) 4W

Explanation: $V = \frac{4}{3}\pi r^3$ or $V \propto r^3$ or $r \propto V^{1/3}$

Now,

$$W = 8\pi r^2 T$$

$$\therefore W \propto r^2 \propto V^{2/3}$$

$$\therefore \frac{W'}{W} = \left(\frac{V'}{V} \right)^2 = \left(\frac{8V}{V} \right)^{2/3} = (8)^{2/3} = 4$$

or $W' = 4W$

10. (c) 8

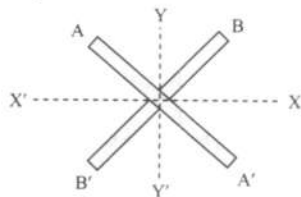
Explanation: $T \propto R^{3/2}$

$$\frac{T_1}{T_2} = \left(\frac{R_1}{R_2} \right)^{3/2}$$

$$\therefore T_2 = T_1(4)^{3/2} = 8T_1$$

11. (a) $\frac{ML^2}{12}$

Explanation:



Moment of inertia of each rod about an axis passing through their centre of mass and perpendicular to length is,

$$I_{ZZ'} = \frac{ML^2}{12}$$

$$\therefore \text{M.I. of cross about ZZ' axes is } 2 \left(\frac{ML^2}{12} \right)$$

$$= \frac{ML^2}{6}$$

Now, consider axes $YY' \perp XX'$

$$\therefore I_{ZZ'} = I_{xx'} + I_{yy'} \dots (\text{perpendicular axes theorem})$$

$$= 2I_{xx'}$$

$$\therefore I_{xx'} = \frac{I_{ZZ'}}{2} = \frac{ML^2}{12}$$

12. (c) 600 K

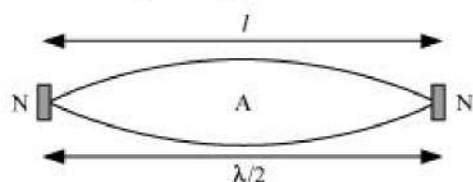
Explanation: 600 K

13. (c) 5.06 km/s

Explanation: Length of the steel rod, $l = 100 \text{ cm} = 1 \text{ m}$

Fundamental frequency of vibration, $\nu = 2.53 \text{ kHz} = 2.53 \times 10^3 \text{ Hz}$

When the rod is plucked at its middle, an antinode (A) is formed at its centre, and nodes (N) are formed at its two ends, as shown in the given figure.



The distance between two successive nodes is $\frac{\lambda}{2}$.

$$\therefore l = \frac{\lambda}{2}$$

$$\lambda = 2l = 2 \times 1 = 2m$$

The speed of sound in steel is given by the relation:

$$v = \nu\lambda$$

$$= 2.53 \times 10^3 \times 2$$

$$= 5.06 \times 10^3 \text{ m/s}$$

$$= 5.06 \text{ km/s}$$

14. (c) $2^{\gamma-1}$

Explanation: The gas in container A is compressed isothermally

$$\therefore P_1V_1 = P_2V_2$$

$$\text{or } P_2 = \frac{P_1V_1}{V_2} = P_1 \frac{V_1}{\frac{V_1}{2}} = 2P_1 \left[\because V_2 = \frac{V_1}{2} \right]$$

Again the gas in container B is compressed adiabatically.

$$\therefore P_1V_1^\gamma = P_2'(V_2')^\gamma$$

$$\text{or } P_2 = P_1 \frac{V_1^\gamma}{(V_2')^\gamma} = P_1 \left[\frac{V_1}{\frac{V_1}{2}} \right]^\gamma$$

$$= 2^\gamma P_1$$

$$\text{Hence, } \frac{P_2'}{P_2} = \frac{2^\gamma P_1}{2P_1} = 2^{\gamma-1}$$

15. (d) $(GMm^2r)^{\frac{1}{2}}$

Explanation: The angular momentum L of a satellite is given by

$$L = mvr \text{ (where, } r = \text{radius of orbit} = R + h)$$

$$= m \left[\sqrt{\left(\frac{GM}{r}\right)} \right] r = (m^2 GMr)^{\frac{1}{2}}$$

16. (c) A is true but R is false.

Explanation: A is true but R is false.

17. (d) A is false but R is true.

Explanation: When a spring balance has been used for a long time, the spring in the balance gets fatigued and there is loss of strength of the spring. In such a case, the extension in the spring is more for a given load and hence the balance gives wrong readings.

18. (a) Both A and R are true and R is the correct explanation of A.

Explanation: Coefficient of viscosity is force acting per unit area per unit velocity gradient i.e.

$$[\eta] = \frac{[F]}{[A] \left[\left(\frac{du}{dr} \right) \right]} = \frac{[MLT^{-2}]}{[L^2][LT^{-1}L^{-1}]} = [ML^{-1}T^{-1}]$$

Section B

19. As $F = G \frac{m_1 m_2}{r^2}$

$$\therefore G = \frac{Fr^2}{m_1 m_2}$$

$$[G] = \frac{MLT^{-2} \cdot L^2}{MM} = M^{-1}L^3T^{-2}$$

$$\therefore a = -1, b = 3, c = -2$$

$$\therefore n_2 = n_1 \left[\frac{M_1}{M_2} \right]^a \left[\frac{L_1}{L_2} \right]^b \left[\frac{T_1}{T_2} \right]^c$$

$$= 6.67 \times 10^{-8} \left[\frac{1}{1000} \right]^{-1} \left[\frac{1}{100} \right]^3 \left[\frac{1}{1} \right]^{-2}$$

Hence in SI units,

$$G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$$

20. Yes. When the direction of motion of a body reverses, the direction of friction is also reversed. Work has to be done against friction both during the forward and return journey i.e., work done against friction along a closed path is not zero. So friction is a non-conservative force.

21. Acceleration due to gravity on the surface of the earth is given by

$$g = \frac{GM}{R^2}$$

Taking logarithm of both sides, we get

$$\log g = \log G + \log M - 2 \log R$$

As G and M are constant, so differentiation of the above equation gives

$$\frac{dg}{g} = 0 + 0 - 2 \frac{dR}{R}$$

As radius of the earth decreases by 2%, so

$$\frac{dR}{R} = -\frac{2}{100}$$

$$\frac{dg}{g} \times 100 = -2 \frac{dR}{R} \times 100$$

$$= -2 \times \left(-\frac{2}{100}\right) \times 100 = 4\%$$

OR

- i. It is a central force.
 - ii. It is a conservation force.
 - iii. It obeys inverse square law.
 - iv. It is a universal force and is always attractive in nature.
 - v. It is an action reaction pair.
 - vi. The gravitational force between two particles is independent of presence or absence of other particles.
 - vii. The work done in moving a particle once around a closed path under the action of gravitational force is zero.
22. Compressibility of the material of a body is defined as the fractional change in volume per unit increase in pressure. It is the reciprocal of its bulk modulus.

Mathematically,

$$\text{Compressibility, } K = \frac{1}{B} = -\left(\frac{\Delta V}{V}\right) \times \frac{1}{p}$$

The solids are least compressible whereas gases are most compressible. It is on account of the fact that in solids neighbouring atoms are tightly coupled but molecules in gases are very poorly coupled to their neighbours.

23. i. The perfect gas equation can be written as, $pV = nRT$

where, p is pressure and n number of moles,

$R = N_A k_B$ is universal constant and T is absolute temperature in Kelvin. We know that,

$$n = \frac{M}{M_0} = \frac{N}{N_A}$$

where, M = mass of the gases

In terms of density, perfect gas equation is

$$p = \frac{\rho RT}{M_0}, \rho = M/V = \text{density of the gas.}$$

- ii. $p = 1 \text{ atm} = 0.76 \text{ m of Hg}$

$$= 0.76 \times (13.6 \times 10^3) \times 9.8 \text{ Pa}$$

$$T = 273 \text{ K, } R = 8.31 \text{ J mol}^{-1}\text{K}^{-1}, n = 1 \text{ mole}$$

We know from ideal gas equation, $pV = nRT$

$$\text{or, } V = \frac{\mu RT}{p} = \frac{1 \times 8.31 \times 273}{0.76 \times (13.6 \times 10^3) \times 9.8}$$

$$= 22.4 \times 10^{-3} \text{ m}^3 = 22.4 \text{ L, this is the required volume by one mole of an ideal gas at NTP.}$$

OR

An ideal gas is one whose molecules have zero volume and no mutual force between them. At low pressure, the volume of a gas is large and so the volume occupied by the molecules is negligible in comparison to the volume of the gas. At high temperature, the molecules have large velocities and so the intermolecular force has no influence on their motion. Hence at low pressure and high temperature, the behaviour of real gases approach the ideal gas behaviour.

24. We know that, $v = \frac{dx}{dt}$

$$v = \frac{d}{dt}(a + bt^2) = 2bt = 5t \text{ m/s(i)}$$

At $t = 0$, from equation (i) we have $v = 0$,

At $t = 2$ from equation (i) we have $v = 10 \text{ m/s}$

$$\text{Average velocity} = \frac{x(4) - x(2)}{4 - 2} = \frac{a + 16b - a - 4b}{2} = 6 \times b = 6 \times 2.5 = 15 \text{ m/s}$$

25. Here $F = -50 \text{ N}$, $m = 30 \text{ kg}$, $u = 18 \text{ ms}^{-1}$ and $v = 0$.

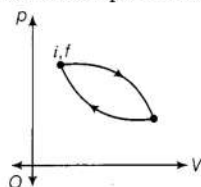
$$\text{Acceleration, } a = \frac{F}{m} = \frac{-50}{30} = \frac{-5}{3} \text{ ms}^{-2}$$

Now from the equation $v = u + at$, we have,

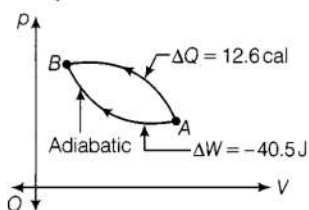
$$t = \frac{v-u}{a} = \frac{0-18}{\left(\frac{-5}{3}\right)} = 10.8 \text{ sec}$$

Section C

26. After completion of the cycle, A cyclic process restores the system back to its initial state after completion of the cycle.



As internal energy is a state function, so its value at initial point is same as that at final point (initial state is same as that of final state).



For adiabatic process $A \rightarrow B$,

$\Delta Q = 0$, Using first law of thermodynamics

$$\Delta Q = \Delta W + \Delta U$$

$$\Delta U = -\Delta W = -(-40.5) = 40.5 \text{ J}$$

For another process, $A \rightarrow B$

$$\Delta Q = +12.6 \text{ cal}$$

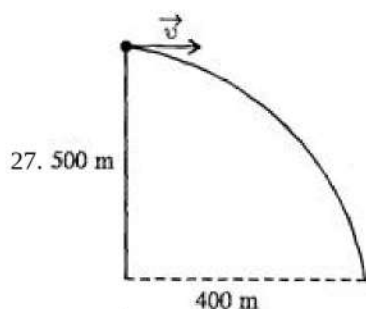
$$= 12.6 \times 4.19 \text{ J} = 52.8 \text{ J}$$

Using the first law of thermodynamics,

$$\Delta Q = \Delta W + \Delta U$$

$$\Rightarrow \Delta W = \Delta Q - \Delta U$$

$$\Rightarrow \Delta W = 52.8 - 40.5 = 12.3 \text{ J}$$



Let the horizontal speed of the ball is $u \text{ ms}^{-1}$ its vertical component will be zero.

Consider the motion of ball vertically downward

Here, mass of ball, $m_b = 1 \text{ kg}$, Mass of gun, $M_G = 100 \text{ kg}$

u_b = initial velocity of ball,

v_b = final velocity of ball,

v_g = final velocity of gun

$$u = 0, s = h = 500 \text{ m}, g = 10 \text{ s}^{-2}$$

$$s = ut + \frac{1}{2}at^2$$

$$500 = 0 \times t + \frac{1}{2} \times 10t^2 \Rightarrow t^2 = \frac{500}{5} = 100$$

$$t = \sqrt{100} = 10 \text{ sec}$$

Horizontal distance covered by the ball is $x = u \times t$

$$400 = v \times 10 \Rightarrow v = 40 \text{ m/sec}$$

By the law of conservation of momentum

$$m_b u_j + M_G u_g = m_b v_b + M_G v_G$$

$$\Rightarrow m_b \times 0 + M_G \times 0 = 1 \times 40 + 100 v_G$$

$$100 v_G = -40$$

Recoil velocity of Gun = $\frac{-40}{100} \text{ ms}^{-1} = \frac{-2}{5} \text{ ms}^{-1} = -0.4 \text{ ms}^{-1}$ i.e opposite to the speed of ball.

28. Energy due to surface Tension $E = \sigma \Delta A$

By law of conservation of mass, volume of drop $V_1 + V_2 = V$

$$r_1 = 0.1 \text{ cm} = 0.1 \times 10^{-2} \text{ m} = 10^{-3} \text{ m}$$

$$r_2 = 0.2 \text{ cm} = 2 \times 10^{-3} \text{ m}$$

$$\Delta A = 4\pi r_1^2 + 4\pi r_2^2 - 4\pi R^2 = 4\pi [r_1^2 + r_2^2 - R^2]$$

R is the radius of new drop formed by the combination of two smaller drops.

$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3$$

$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi [r_1^3 + r_2^3] \Rightarrow R^3 = r_1^3 + r_2^3$$

$$R^3 = [(1 \times 10^{-3})^3 + (2 \times 10^{-3})^3] = [10^{-9} + 8 \times 10^{-9}] = 9 \times 10^{-9}$$

$$R = 2.1 \times 10^{-3} \text{ m}$$

$$E = \Delta A \sigma = 4 \times 3.14 [(10^{-3})^2 + (2.0 \times 10^{-3})^2$$

$$- (2.1 \times 10^{-3})^2] \times 435.5 \times 10^{-3}$$

$$E = 4 \times 3.14 \times 435.5 \times 10^{-3} \times (10^{-3})^2 [1 + 4 - (2.1)^2]$$

$$= 4 \times 3.14 \times 435.5 \times 10^{-9} [5 - 4.41];$$

$$E = 1742.0 \times 3.14 \times 10^{-9} [0.59] = 5469.88 \times 0.59 \times 10^{-9}$$

$$E = 3227.23 \times 10^{-9} = 32.2723 \times 10^{-7} \text{ J}$$

$$E = 32.27 \times 10^{-7} \text{ J}$$

Energy is released due to formation of bigger drop from smaller drops because final area will be smaller than former case.

OR

Venturi-meter is a device used to measure the flow speed of a liquid. It is basically based on Bernoulli's principle and works on the principle that when a liquid flows in the tube from wide neck to a narrow constriction, the speed of flow increases and the pressure falls.

Bernoulli's principle states that with the increase in the velocity of the fluid its pressure decreases (or) there is a decrease in the fluid pressure energy. This decrease in the fluid pressure in the areas where the flow velocity is increased is called the Bernoulli effect.

It is utilised in the carburettor of automobiles. The carburettor has a venturi channel (fine nozzle) through which air flows with a large speed. The pressure is then lowered at the narrow neck as a result, the valve of petrol chambers opens and the petrol is sucked up in the chamber to provide the correct mixture of air and petrol necessary for combustion.

29. Given, $y_1 = 5 \sin (4x - t)$ cm

$$y_2 = 5 \sin (4x + t) \text{ cm}$$

We know that the resulting wave equation,

$$y = (2A \sin kx) \cos \omega t$$

Now, comparing the given equation in the question with the above resulting wave equation we get,

$$y_1 = 5 \sin (4x - t) \text{ cm with } y_1 = A \sin (kx - \omega t).$$

$$A = 5 \text{ cm, } k = 4 \text{ cm}^{-1} \text{ and } \omega = 1 \text{ rad/s}$$

Hence, $y = (2A \sin kx) \cos \omega t$ becomes

$$y = (10 \sin 4x) \cos t$$

The maximum displacement of the motion at position

$$x = 2.0 \text{ cm equals to}$$

$$y_{\max} = 10 \sin 4x, x = 2.0 \text{ and } t = 0$$

$$= 10 \sin (4 \times 2) = 10 \sin (8 \text{ rad})$$

$$\Rightarrow y_{\max} = 9.89 \text{ cm}$$

The wavelength by using the relation between wavelength and wave number, we get

$$k = \frac{2\pi}{\lambda} = 4$$

$$\Rightarrow \lambda = \frac{2\pi}{4} = \frac{\pi}{2} \text{ cm}$$

The nodes and antinodes can be given as

$$\text{Nodes at } x = \frac{n\lambda}{2} = n \times \left(\frac{\pi}{4}\right) \text{ cm,}$$

where $n = 0, 1, 2, \dots$ any integer

$$\text{Antinodes at } x = (2n + 1) \frac{\lambda}{4} = (2n + 1) \times \left(\frac{\pi}{8}\right) \text{ cm,}$$

where $n = 0, 1, 2, \dots$ any integer

OR

Let v_S and v_P be the velocities of S(Secondary wave) and P(Primary Wave) waves respectively.

Let L be the distance between the epicenter and the seismograph.

We have:

$$L = v_S t_S \dots(i)$$

$$L = v_P t_P \dots(ii)$$

Where,

t_S and t_P are the respective times taken by the S and P waves to reach the seismograph from the epicentre

It is given that:

$$v_P = 8 \text{ km/s}$$

$$v_S = 4 \text{ km/s}$$

From equations (i) and (ii), we have:

$$v_S t_S = v_P t_P$$

$$4 \times t_S = 8 \times t_P$$

$$t_S = 2t_P \dots(iii)$$

It is also given that:

$$t_S - t_P = 4 \text{ min} = 240 \text{ s}$$

from the equation (iii), we get $2t_P - t_P = 240$

$$t_P = 240 \text{ sec}$$

so time taken by P wave to reach from epicenter to seismograph is 240 sec and we know that $t_S = 2t_P$

$$t_S = 2 \times 240 = 480 \text{ sec}$$

From equation (ii), we get:

$$L = 8 \times 240$$

$$= 1920 \text{ km}$$

Hence, distance of earthquake from epicenter is 1920 km.

30. Power of the drilling machine, $P = 10 \text{ kW} = 10 \times 10^3 \text{ W} = 10^4 \text{ W}$

Mass of the aluminum block, $m = 8.0 \text{ kg} = 8000 \text{ g}$

Time, $t = 2.5 \text{ min} = 2.5 \times 60 = 150 \text{ s}$

Specific heat of aluminium, $c = 0.91 \text{ J g}^{-1}\text{K}^{-1}$

Let rise in the temperature of the block after drilling = δT

Total energy of the drilling machine = $P \times T$

$$= 10 \times 10^3 \times 150 = 1.5 \times 10^6 \text{ J}$$

As only 50% of the energy is useful as per the question

so useful energy, $\Delta Q = \frac{50}{100} \times 1.5 \times 10^6 = 7.5 \times 10^5 \text{ J}$

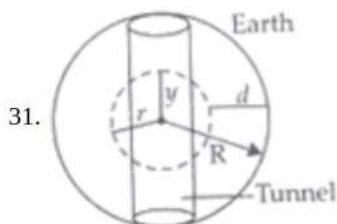
But $\Delta Q = mc\Delta T$

$$\therefore \Delta T = \frac{\Delta Q}{mc}$$

$$= \frac{7.5 \times 10^5}{8 \times 10^3 \times 0.91} = 103^\circ\text{C}$$

Therefore, temperature of block increases by 103°C in drilling for 2.5 minutes.

Section D



If the acceleration due to gravity of earth inside the earth is at a depth of d is g' , then we know that,

$$g' = g \left(1 - \frac{d}{R}\right) = g \left[\frac{R-d}{R}\right] \dots(i) \text{ (} g = \text{acceleration due to gravity on the surface of the earth, } R = \text{Radius of earth)}$$

Now if 'y' be distance of the point where acceleration due to gravity is g' from the centre of the earth, then $R - d = y$, and from equation (i) we get,

$$\therefore g' = g \frac{y}{R}$$

Force on the body of mass m placed at depth d from the surface of the earth is

$$F = -mg' = -mg \frac{y}{R}$$

$F \propto (-y)$, i.e. the force is proportional to displacement but opposite to the direction of displacement.

So motion of body in tunnel is SHM.

Now to get the time period of this simple harmonic motion we can write,

$$ma = -mg'$$

$$\Rightarrow a = -\frac{g}{R}y$$

$$\therefore -\omega^2 y = -\frac{g}{R}y \quad (\because a = -\omega^2 y)$$

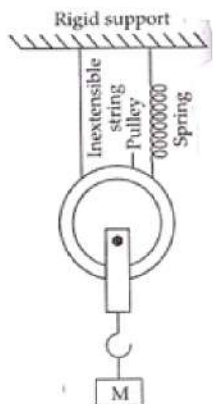
$$\therefore \frac{2\pi}{T} = \sqrt{\frac{g}{R}} \text{ or } T = 2\pi \sqrt{\frac{R}{g}}$$

This is the time period of the simple harmonic motion executed by the body.

OR

When mass M is pulled and released then, mass M oscillates up and down along with pulley

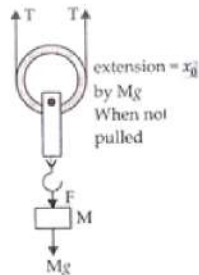
Let the spring extends by x_0 when loaded by the mass M . The extension and compression of spring from initial position is larger and smaller respectively due to gravity by same amount of forces always. So effect of gravitational force can be neglected here.



Now let the mass 'M' is pulled by force 'F' downward by displacement x . Then extension in spring will be $2x$ as string can not be extended.

So, total extension in spring is given by $(x_0 + 2x)$

$T' = k(x_0 + 2x)$ (when pulled downward by x)



$T = kx_0$ (when no pulling)

$$F = 2T$$

$$F = 2kx_0$$

And,

$$F' = 2T'$$

$$\rightarrow F' = 2k(x_0 + 2x)$$

Restoring force is given by ;

$$\rightarrow F_{rest} = -(F' - F)$$

$$\rightarrow f_{rest} = -[2k(x_0 + 2x) - 2kx_0]$$

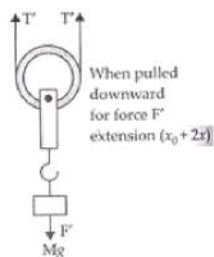
$$\rightarrow F_{rest} = -2k \cdot 2x$$

$$\rightarrow Ma = -4kx$$

$$a \propto -x$$

\rightarrow Hence, the motion is said to be simple harmonic motion(SHM).





$$\rightarrow a = -\omega^2 x$$

$$\therefore \omega^2 = \frac{-a}{x} = \frac{+4k}{M}$$

$$\omega = 2\sqrt{\frac{k}{M}}$$

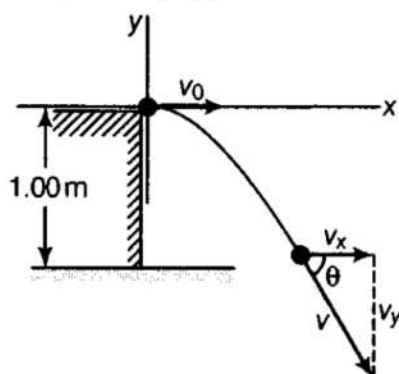
$$\Rightarrow \frac{2\pi}{T} = 2\sqrt{\frac{k}{M}}$$

$$T = \pi\sqrt{\frac{M}{k}}$$

→ Therefore, Time period is given by the relation → $T = \pi\sqrt{\frac{M}{k}}$

32. As the marble was rolling on the table, therefore it has horizontal velocity and it will act as a projectile as soon as it leaves the edge of the table and fall freely under the effect of gravity.

Since, the marble is initially moving horizontally, $v_{y0} = 0$ and $v_{x0} = 1.00$ m/s. We must consider the origin to be at the edge of the table, so that $x_0 = y_0 = 0$



i. $t = ?$ and $y = -1.00$ m

$$\therefore y = \frac{-1}{2}gt^2$$

$$\Rightarrow t = \sqrt{\frac{-2y}{g}} = \sqrt{\frac{(-2)(-1.00)}{9.8}} = 0.452 \text{ s}$$

ii. $x = ?$, when $t = 0.452$ s

$$\therefore x = v_{x0}t = 1.00 \times 0.452 \text{ s} = 0.452 \text{ m}$$

iii. $v = ?$, $\theta = ?$ at $t = 0.452$ s

The x-component of velocity is constant throughout the motion,

$$v_x = v_{x0} = 1.00 \text{ m/s}$$

The y-component of velocity is given by

$$v_y = v_{y0} - gt = 0 - 9.8 \times 0.452 = -4.43 \text{ m/s}$$

$$\therefore v = \sqrt{v_x^2 + v_y^2} = \sqrt{(1.00)^2 + (-4.43)^2} = 4.54 \text{ m/s, the magnitude of the resultant velocity of the motion.}$$

$$\theta = \tan^{-1} \left| \frac{v_y}{v_x} \right| = \frac{4.43}{1.00} = 77.3^\circ$$

As the marble hits the floor, its velocity is 4.54 m/s directed 77.3° downward with respect to the horizontal.

OR

Maximum range of projectile, $R = \frac{V_0^2}{g}$ (i)

We know, maximum range is achieved at $\theta = 45^\circ$

Let the gun is raised to a height h from the horizontal level of target T, so that the projectile can hit the target T.

Total range of projectile must be $R_{\text{total}} = R + \Delta x$

Horizontal component of velocity at A = $v_0 \cos(\theta)$

As A and P are on the same level, the magnitude of velocity will be the same at A and P.

But the direction of velocity will be below horizontal,

So horizontal velocity at P, $v_x = -v_0 \cos(\theta)$

and vertical velocity at P, $v_y = v_0 \sin(\theta)$

Now $h = ut + \frac{1}{2}at^2$

$$h = -v_0 \sin \theta(t) + \frac{1}{2}gt^2 \dots\dots(ii)$$

Consider horizontal motion from A to T, distance $(R + \Delta x) = v_0 \cos \theta \cdot t$

$$t = \frac{R + \Delta x}{v_0 \cos \theta}$$

Substitute t in (ii) we get

$$h = -v_0 \sin \theta \left[\frac{R + \Delta x}{v_0 \cos \theta} \right] + \frac{1}{2}g \frac{(R + \Delta x)^2}{v_0^2 \cos^2 \theta}$$

$$h = -\tan \theta (R + \Delta x) + \frac{1}{2} \left(\frac{g}{v_0^2} \right) \frac{(R + \Delta x)^2}{1/2} \quad (\because \theta = 45^\circ \Rightarrow \tan(45) = 1)$$

$$h = -(R + \Delta x) + \frac{1}{R} (R^2 + \Delta x^2 + 2R\Delta x) \left[\because \frac{g}{v_0^2} = \frac{1}{R} \right]$$

$$h = -R - \Delta x + R + \frac{\Delta x^2}{R} + 2\Delta x = \Delta x + \frac{\Delta x^2}{R}$$

$$h = \Delta x \left[1 + \frac{\Delta x}{R} \right]$$

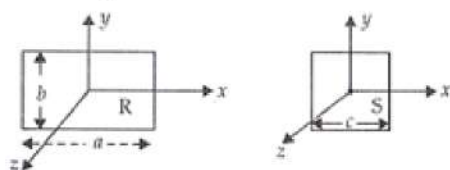
Hence Proved.

33. Moment of inertia, in physics, quantitative measure of the rotational inertia of a body- i.e., the opposition that the body exhibits to having its speed of rotation about an axis altered by the application of a torque (turning force). The axis may be internal or external and may or may not be fixed. The moment of inertia (I), however, is always specified with respect to that axis and is defined as the sum of the products obtained by multiplying the mass of each particle of matter in a given body by the square of its distance from the axis. The unit of moment of inertia is a composite unit of measure. In the International System (SI), m is expressed in kilograms and r in metres, with I (moment of inertia) having the dimension kilogram-metre square.

$$m_R = m_S = m$$

Area of square = Area of rectangle

$$c^2 = ab \dots(i)$$



a. $\because I = mr^2$

$$\frac{I_{xR}}{I_{xz}} = \frac{m \cdot \left(\frac{b}{2}\right)^2}{m \left(\frac{c}{2}\right)^2} = \frac{b^2}{4} \cdot \frac{4}{c^2} = \frac{b^2}{c^2}$$

$\because c > b$ [from (i)]

Or $c^2 > b^2$

$$1 > \frac{b^2}{c^2} : \frac{I_{xR}}{I_{xz}} < 1$$

Hence proved.

b. $\frac{I_{yR}}{I_{ys}} = \frac{m \left(\frac{a}{2}\right)^2}{m \left(\frac{c}{2}\right)^2} = \frac{a^2}{4} \cdot \frac{4}{c^2} = \frac{a^2}{c^2}$

$\because a > c \Rightarrow \frac{a^2}{c^2} > 1$

$$\frac{I_{yR}}{I_{ys}} > 1$$

c. $I_{zR} - I_{zs} = m \left(\frac{d_R}{2}\right)^2 - m \left(\frac{d_S}{2}\right)^2$

$$I_{zR} - I_{zs} = \frac{m}{4} [d_R^2 - d_S^2] = \frac{m}{4} [a^2 + b^2 - 2c^2]$$

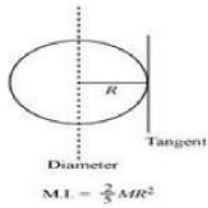
$$\therefore I_{zR} - I_{zs} = \frac{m}{4} (a^2 + b^2 - 2ab) = \frac{m}{4} (a - b)^2 \quad (c^2 = ab)$$

$$\therefore I_{zR} - I_{zs} > 0 \because \frac{m}{4} (a - b)^2 > 0$$

$$\Rightarrow \frac{I_{zR}}{I_{zs}} > 1 \text{ Hence proved.}$$

OR

a. The moment of inertia (M.I.) of a sphere about its diameter is given by $= \frac{2}{5}MR^2$



Given,

Moment of inertia of the sphere about its diameter is given by $= (\frac{2}{5})mR^2$

Use, parallel axis theorem,

Moment of inertia of the sphere about tangent is given by $= I + mR^2$

$$= (\frac{2}{5})mR^2 + mR^2$$

$$= (7/5)mR^2$$

b. Moment of inertia of disc of mass m and radius R about any of its diameter is $= mR^2/4$

Moment of inertia about diameter is given by $= I_x = I_y = (\frac{1}{4})mR^2$

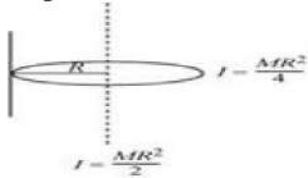
Using, perpendicular axis theorem,

$$I_z = I_x + I_y$$

Where I_z is moment of inertia about perpendicular axis of plane of disc. Hence,

$$I_z = (\frac{1}{4})mR^2 + (\frac{1}{4})mR^2$$

$$= (\frac{1}{2})mR^2$$



Moment of inertia of disc about passing through a point of its edge is given by;

Use, parallel axis theorem, we get

$$I = I_z + mR^2$$

$$= (\frac{1}{2})mR^2 + mR^2$$

$$= (\frac{3}{2})mR^2$$

Section E

34. Read the text carefully and answer the questions:

Free fall is a kind of motion that everybody can observe in daily life. We drop something accidentally or purposely and see its motion. At the beginning its speed is zero and until the end it gains speed and before it reaches ground its maximum speed. It gains speed approximately 10 m/s in a second while falling because of the gravitation.

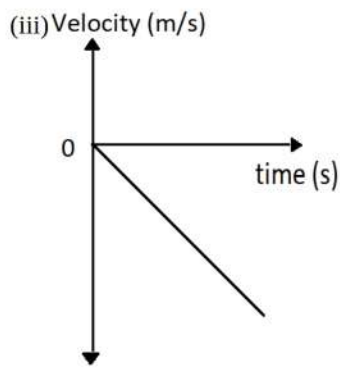
During the fall, the air resistance is neglected and the acceleration remains constant (equal to g). The object is said to be in free fall. If the height through which the object falls is small compared to the earth's radius, g can be taken to be constant and equal to 10 m/s² approximately.

If the object is dropped from the top of a tall building, and it takes t seconds to reach the ground then the velocity when it reaches ground is gt . The height of the building is $\frac{1}{2}gt^2$.

(i) Height of building is given as

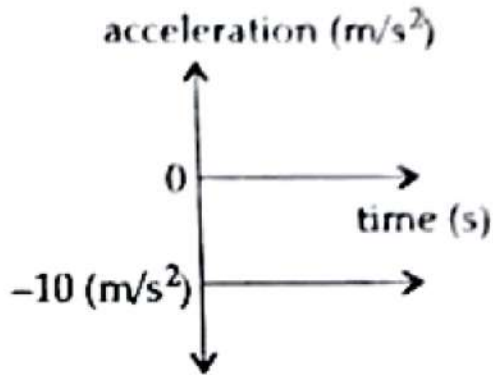
$$H = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times 2^2 = 20 \text{ m}$$

(ii) It is assumed that there is no air resistance and height through which the object falls is small compared to the earth's radius.



OR

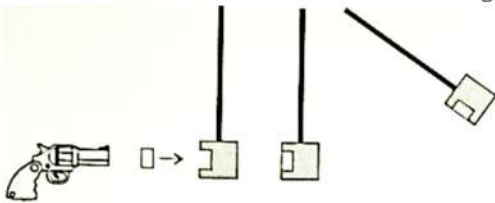
The acceleration is constant during the free fall, acceleration time graph is given as



35. Read the text carefully and answer the questions:

The ballistic pendulum was invented in 1742 by English mathematician Benjamin Robins.

A Ballistic Pendulum is a device for measuring a bullet's momentum and speed by employing perfectly inelastic collision.



A large wooden block suspended by two cords serves as the pendulum bob. When a bullet is fired into the bob, it gets embedded in the bob and its momentum is transferred to the bob.

The bullet's momentum and velocity can be determined from the amplitude of the pendulum swing. The velocity of the bullet, in turn, can be derived from its calculated momentum.

After collision, if the pendulum reaches a height h , then from principle of conservation of mechanical energy

$$\frac{1}{2}(m + M)v_p^2 = (m + M)gh$$

where, m = mass of bullet, M = mass of the bob v_p = velocity of the bob-bullet combination

$$\therefore v_p = \sqrt{2gh}$$

Now, Momentum before collision = Momentum after collision

$$mv_B = (m + M)v_p$$

where, v_B = velocity of bullet

$$v_B = \frac{m+M}{m} \sqrt{2gh}$$

the ballistic pendulum used to be a common tool for the determination of the muzzle velocity of bullets as a measure of the performance of firearms and ammunition (Nowadays, the ballistics pendulum has been replaced by the ballistic chronograph, an electronic device).

(i) Perfectly inelastic takes place in ballistic pendulum. A large wooden block suspended by two cords serves as the pendulum bob.

When a bullet is fired into the bob, it gets embedded in the bob and its momentum is transferred to the bob. Hence the collision is perfectly inelastic.

(ii) Conservation of mechanical energy and conservation of momentum.

Principle of conservation of mechanical energy, an expression for the bob-bullet combination after collision is derived.

Then the principle of conservation of momentum is applied to find the velocity of the bullet before collision.

(iii) The ballistic pendulum has now been replaced by the ballistic chronograph, an electronic device.

OR

$$v_B = \frac{m+M}{m} \sqrt{2gh}$$

Putting, $m = 1\text{g} = 0.001\text{ kg}$

$M = 1\text{ kg}$

$g = 10\text{ m/s}^2$

$h = 2\text{ cm} = 0.02\text{ m}$

$$v_B = \frac{0.001+1}{0.001} \sqrt{2 \times 10 \times 0.02} = 633\text{ m/s}$$